On Fuzzy Neutrosophic Supra Soft Topological Spaces

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Abstract: In this paper, we introduce the fuzzy neutrosophic supra soft topological space and we define the notions of fuzzy neutrosophic soft closure, fuzzy neutrosophic soft interiorin fuzzy neutrosophic supra soft topological space with examples and some of their properties are investigated.

Keywords: Fuzzy neutrosophic soft set (FNSS), fuzzy neutrosophic supra soft topological space (FNSSTS), fuzzy neutrosophic soft interior (FNSI), fuzzy neutrosophic soft closure (FNSC).

I. INTRODUCTION

The introduction of the concept of fuzzy sets by Zadeh [13]. Chang [4] introduced the concept of fuzzy topological spaces, the concept of intuitionistic fuzzy sets was introduced by k. Atanassov [12]. Coker [5] introduced intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. Mashhour et al. [2] introduced the concepts of supra topological spaces, supra closed sets, supra open sets. Later El monsef and Ramadan [14] introduced a fuzzy supra topological spaces, Abbas [17] introduced the intuitionistic supra fuzzy topological spaces.Molodtsov [6,8,9] introduced the concept of soft set theory which is a completely new approach for modelling uncertainty and also he introduced some different applications of soft sets and fuzzy soft sets in topology. Shabir and Naz [15] introduced the study of soft topological spaces and S.A.El-sheikh and A.M.Abd-El-Latif [14] introduced the supra soft topological spaces. Cigdem Gunduz Aras [3] introduced a study on intuitionistic fuzzy soft supra topological spaces.

The concept of neutrosophic set was introduced by smarandache [1,7]. Later Naji [10,11] has introduced concept of neutrosophic soft sets. Tuhin Bera [19] introduced the concept of neutrosophic soft topological space.

In this paper, we introduce the fuzzy neutrosophic supra soft topological spacesand we define the notions of fuzzy neutrosophic soft closure, fuzzy neutrosophic soft interiorin fuzzy neutrosophic supra soft topological space with examples and some of their properties are investigated.

II. PRELIMINARIES

Definition 1.1[7]: Let X be a space of points with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth membership function T_A , an indeterminacy I_A and a falsity membership function F_A. $T_A(x), I_A(x)$ *and* $F_A(x)$ are real standard or non - standard subsets of $]^{-}0, 1^{+}[$. That is $T_A, I_A, F_A: X \to]^{-}0, 1^{+}[$. There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$ and so, $^{-}0 \leq \sup T_A(x) + sup I_A(x) + sup\ F_A(x) \geq 3^{+}$.

Definition 1.2[6]: Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F: A \to NS(U)$ is a mapping.

Definition 1.3[16]: Let U be an initial universe set and E be a set of parameters. Let FNS(U) denote the set of all FNSs of U. Then a fuzzy neutrosophic soft set N over U is a set defined be a set valued function f_N representing a mapping $f_N: E \to NS(U)$ where $f_N$is called approximate function of the fuzzy neutrosophic soft set N. In other words, the fuzzy neutrosophic soft set is a parameterized family of some elements of the set FNS(U) and it can be written as a set of ordered pairs,

$$FN = \{(e, \{< x, T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) >: x \in U\}): e \in E\}$$

Where $T_{fN(e)}(x), I_{fN(e)}(x), F_{fN(e)}(x) \in [0,1]$ respectively called the truth membership, indeterminacy, falsity membership function of $f_N(e)$. Since supremum of each T, I, F is 1 so the inequality$0 \leq T_{fN(e)}(x) + I_{fN(e)}(x) + F_{fN(e)}(x) \leq 3$ is obvious.

Example 1.4: Let $U = \{p, q, r\}$ be a set of houses and $E = \{e_1(cement), e_2(wooden),\ e_3(iron)\}$ be a set of parameters.
Let $f_N(e_1) = \{< p, 0.2, 0.5, 0.8 >< q, 0.3,0.5,0.7 >< 0.4,0.5,0.6 >\}$
$f_N(e_2) = \{< p, 0.3, 0.5, 0.7 >< q, 0.4,0.5,0.6 >< 0.5,0.5,0.4 >\}$
$f_N(e_3) = \{< p, 0.4, 0.5, 0.6 >< q, 0.5,0.5,0.4 >< 0.6,0.5,0.4 >\}$
Then $N = \{(e_1, f_N(e_1)), (e_2, f_N(e_2)), (e_3, f_N(e_3))\}$ is a fuzzy neutrosophic soft set over (U, E).

Definition 1.5[7,16]: For two fuzzy neutrosophic soft sets (U, E) *and* (V, E)over a common universe X with parameter E, we say that $(U, E)$is fuzzy neutrosophic soft subset of (V, E) and write $(U, E) \subseteq (V, E)$ if$T_U(e) \leq T_V(e), I_U(e) \leq I_V(e), F_U(e) \geq F_V(e)$.

Definition 1.6[7,16]: The union of two fuzzy neutrosophic soft sets(U,E) and (V,E)over a common universe X with parameter E is the fuzzy neutrosophic soft set $N = \{T_U(e) \cup T_V(e), I_U(e) \cup I_V(e), F_U(e) \cap F_V(e)\}$ for all $e \in E$. It is written as $N = (U,E) \cup (V,E)$

Definition 1.7[7,16]: The intersection of two fuzzy neutrosophic soft sets (U,E) and (V,E) over a common universe X with parameter E is the fuzzy neutrosophic soft set $N = \{T_U(e) \cap T_V(e), I_U(e) \cap I_V(e), F_U(e) \cup F_V(e)\}$ for all $e \in E$. It is written as $N = (U,E) \cap (V,E)$.

Definition 1.8[7,16]: The complement of a fuzzy neutrosophic soft set N is denoted by N^C and is defined by $N^C = \{(e, \{x, 1 - T_{fN(e)}(x), 1 - I_{fN(e)}(x), 1 - F_{fN(e)}(x)\})\}$.

Definition 1.9[18,21]: Let NSS(U,E) be a family of all neutrosophic soft sets over U via parameters in E and $\tau_u \subset NSS(U,E)$. Then τ_u is called neutrosophic soft topology on (U,E) if the following conditions are satisfies:

(i) $\emptyset_u, 1_u \in \tau_u$

(ii) The intersection of any finite number of members of τ_u also belongs to τ_u.

(iii) The union of any collection of members of τ_u belongs to τ_u

Then the pair (U, E, τ_u) is called the neutrosophic soft topology.

III. FUZZY NEUTROSOPHIC SUPRA SOFT TOPOLOGICAL SPACES

Definition 2.1: Let fuzzy neutrosophic soft set (X,E) be the family of all fuzzy neutrosophic soft sets over X with parameters in E and $\tau_{FN} \subset FNSS(X,E)$.Then τ_{FN} is called fuzzy neutrosophic supra soft topology on (X,E) if the following conditions are satisfied:

(i) $0_{FN}, 1_{FN} \in \tau_{FN}$

(ii) The union of any collection of members of τ_{FN} belongs to τ_{FN}

Then the pair (X, E, τ_{FN}) is called a fuzzy neutrosophic supra soft topological space.

The element of τ_{FN} is called $\tau_{FN} -$ fuzzy neutrosophic supra soft open set (FNSSOS) and the complement of τ_{FN} is called fuzzy neutrosophic supra soft closed set (FNSSCS).

Example 2.2: Let $X = \{p,q\}, E = \{e_1, e_2\}$ and consider the family $\tau_{FN} = \{0_{FN}, (F_1,G_1), (F_2,G_2), (F_3,G_3), 1_{FN}\}$ where $(F_1,G_1), (F_2,G_2), (F_3,G_3)$ are fuzzy neutrosophic soft sets $(F_i,G_i): E \to FNS(X)$ on X are defined as follows:

$(F_1,G_1)(e_1) = \{\langle p, 0.1,0.5,0.9\rangle\langle q, 0.2,0.5,0.8\rangle\}$

$(F_1,G_1)(e_2) = \{\langle p, 0.3,0.5,0.7\rangle\langle q, 0.4,0.5,0.6\rangle\}$

$(F_2,G_2)(e_1) = \{\langle p, 0.2,0.5,0.8\rangle\langle q, 0.3,0.5,0.7\rangle\}$

$(F_2,G_2)(e_2) = \{\langle p, 0.4,0.5,0.6\rangle\langle q, 0.5,0.5,0.6\rangle\}$

$(F_3,G_3)(e_1) = \{\langle p, 0.4,0.5,0.6\rangle\langle q, 0.5,0.5,0.4\rangle\}$

$(F_3,G_3)(e_2) = \{\langle p, 0.6,0.5,0.4\rangle\langle q, 0.7,0.5,0.3\rangle\}$ are FNSSOSs.

Then the pair (X, E, τ_{FN}) is called fuzzy neutrosophic supra soft topological space.

Definition 2.3: Let (X, E, τ_{FN}) be a fuzzy neutrosophic supra soft topological space over (X,E) and $P \in FNS(X,E)$ be arbitrary. Then the interior of P is denoted by P° and it is defined as $P^\circ = \cup \{M: M\ is\ FNSSOS\ and\ M \subset P\}$.

i.e, It is the union of all open fuzzy neutrosophic supra soft subsets of P.

Definition 2.3: Let (X, E, τ_{FN}) be a fuzzy neutrosophic supra soft topological space over (X,E) and $P \in FNS(X,E)$ be arbitrary. Then the closure of P is denoted by $P^\otimes$ and it is defined as $P^\otimes = \cap\{N: N\ is\ FNSSCS\ and\ N \supset P\}$.

i.e, It is the intersection of all closed fuzzy neutrosophic supra soft super sets of P.

Proposition 2.4: Let (X, E, τ_{FN}) and (X, E, δ_{FN}) be two fuzzy neutrosophic supra soft topological spaces over X. Then $(X, E, \tau_{FN} \cap \delta_{FN})$ is a fuzzy neutrosophic supra soft topological space over X.

Remark 2.5: The union of two fuzzy neutrosophic supra soft topologies on X may not be fuzzy neutrosophic supra soft topology on X.

Example 2.6: Let $X = \{p,q\}, E = \{e_1, e_2\}$ and $\tau_{FN} = \{0_{FN}, (F_1,G_1), (F_2,G_2), 1_{FN}\}$, $\delta_{FN} = \{0_{FN}, (M_1,N_1), (M_2,N_2), 1_{FN}\}$ where $(F_1,G_1), (F_2,G_2), (M_1,N_1), (M_2,N_2)$ are fuzzy neutrosophic soft sets on X are defined as follows:

$(F_1,G_1)(e_1) = \{\langle p, 0.35,0.5,0.65\rangle\langle q, 0.45,0.5,0.55\rangle\}$

$(F_1,G_1)(e_2) = \{\langle p, 0.3,0.5,0.7\rangle\langle q, 0.4,0.5,0.6\rangle\}$

$(F_2,G_2)(e_1) = \{\langle p, 0.5,0.5,0.4\rangle\langle q, 0.6,0.5,0.4\rangle\}$

$(F_2,G_2)(e_2) = \{\langle p, 0.4,0.5,0.6\rangle\langle q, 0.5,0.5,0.6\rangle\}$

$(M_1,N_1)(e_1) = \{\langle p, 0.65,0.5,0.35\rangle\langle q, 0.55,0.5,0.45\rangle\}$

$(M_1,N_1)(e_2) = \{\langle p, 0.45,0.5,0.5\rangle\langle q, 0.6,0.5,0.4\rangle\}$

$(M_2,N_2)(e_1) = \{\langle p, 0.5,0.5,0.4\rangle\langle q, 0.4,0.5,0.6\rangle\}$

$(M_2,N_2)(e_2) = \{\langle p, 0.4,0.5,0.6\rangle\langle q, 0.5,0.5,0.6\rangle\}$ are FNSSOSs.

Then (X, E, τ_{FN}) and (X, E, δ_{FN}) are two fuzzy neutrosophic supra soft topological spaces.

Here $(F_1,G_1) \cap (M_1,N_1) \in \tau_{FN} \cap \delta_{FN}$ but $(F_2,G_2) \cup (M_2,N_2) \notin \tau_{FN} \cup \delta_{FN}$.

Therefore $\tau_{FN} \cap \delta_{FN}$ is FNSSTS but $\tau_{FN} \cup \delta_{FN}$ is not FNSSTS.

Theorem 2.8: Let (X, E, τ_{FN}) be a FNSSTS over (X, E) and $M, N \in FNSS(X, E)$. Then

(i) $M^{\circ} \subseteq M$ and M° is the largest FNSSOS

(ii) $M \subset N \; implies \; M^{\circ} \subset N^{\circ}$

(iii) M° is FNSSOS, i.e $M^{\circ} \in \tau_{FN}$

(iv) M is FNSSOS $\Leftrightarrow M^{\circ} = M$

(v) $(M^{\circ})^{\circ} = M^{\circ}$

(vi) $(0_{FN})^{\circ} = 0_{FN}$ and $(1_{FN})^{\circ} = 1_{FN}$

(vii) $(M \cap N)^{\circ} = M^{\circ} \cap N^{\circ}$

(viii) $M^{\circ} \cup N^{\circ} = (M \cup N)^{\circ}$

Proof: Obvious

Theorem 2.9: Let (X, E, τ_{FN}) be a FNSSTS over (X, E) and $M, N \in FNSS(X, E)$. Then

(i) $M \subset M^{\otimes}$ and $M^{\otimes}$ is the smallest FNSSCS

(ii) $M \subset N$ implies $M^{\otimes} \subset N^{\otimes}$

(iii) $M^{\otimes}$ is FNSSCS, i.e $M^{\otimes} \in \tau_{FN}^{c}$

(iv) M is FNSSCS $\Leftrightarrow M^{\otimes} = M$

(v) $(M^{\otimes})^{\otimes} = M^{\otimes}$

(vi) $(0_{FN})^{\otimes} = 0_{FN}$ and $(1_{FN})^{\otimes} = 1_{FN}$

(vii) $(M \cap N)^{\otimes} \subset M^{\otimes} \cap N^{\otimes}$

(viii) $M^{\otimes} \cup N^{\otimes} = (M \cup N)^{\otimes}$

Proof: Obvious

Theorem 2.10: Let (X, E, τ_{FN}) be a FNSSTS over (X, E) and $N \in FNSS(X, E)$. Then

(i) $(M^{\otimes})^{C} = (M^{C})^{\circ}$

(ii) $(M^{\circ})^{C} = (M^{C})^{\otimes}$

Proof: Obvious

IV. CONCLUSION

Topology is a major sector in mathematics and it can give many relationships between other scientific area and mathematical models. The motivation of the present paper is to extend the concept of intuitionistic fuzzy soft supra topological into fuzzy neutrosophic supra soft topolohical space. Here we defined fuzzy neutrosophic supra soft interior and fuzzy neutrosophic supra soft closuresome examples and their properties are investigated.

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